This is 369 Tesla Formula Sheet

Below is a comprehensive "formula sheet" for the material shown in your three images, written out in English and including a simple example for each formula. The sheet is organized in sections:

1. Basic Derivatives and Their Corresponding Integrals
2. Integration by Substitution (Selected Standard Results)
3. Integrals of Particular Forms
4. Rational-Function Integrals and Partial Fractions
5. Integration by Parts
6. Special Composite Forms
7. Fundamental Theorems of Calculus
8. Definite Integrals
9. Properties of Definite Integrals

1) Basic Derivatives and Their Corresponding Integrals

Derivative	Integral (Antiderivative)	Example
$\dfrac{d}{dx}(x^n) = n\,x^{n-1}$	$\displaystyle\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad (n \neq -1)$	$\int x^2\,dx = \frac{x^3}{3} + C.$
$\dfrac{d}{dx}(x) = 1$	$\displaystyle\int 1\,dx = x + C$	$\int dx = x + C.$
$\dfrac{d}{dx}(\sin x) = \cos x$	$\displaystyle\int \cos x\,dx = \sin x + C$	$\int \cos x\,dx = \sin x + C.$
$\dfrac{d}{dx}(\cos x) = -\sin x$	$\displaystyle\int \sin x\,dx = -\cos x + C$	$\int \sin x\,dx = -\cos x + C.$
$\dfrac{d}{dx}(\tan x) = \sec^2 x$	$\displaystyle\int \sec^2 x\,dx = \tan x + C$	$\int \sec^2 x\,dx = \tan x + C.$
$\dfrac{d}{dx}(\cot x) = -\csc^2 x$	$\displaystyle\int \csc^2 x\,dx = -\cot x + C$	$\int \csc^2 x\,dx = -\cot x + C.$
$\dfrac{d}{dx}(\sec x) = \sec x \tan x$	$\displaystyle\int \sec x \tan x\,dx = \sec x + C$	$\int \sec x \tan x\,dx = \sec x + C.$
$\dfrac{d}{dx}(\csc x) = -\csc x \cot x$	$\displaystyle\int \csc x \cot x\,dx = -\csc x + C$	$\int \csc x \cot x\,dx = -\csc x + C.$
$\dfrac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$	$\displaystyle\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$	$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C.$
$\dfrac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$	$\displaystyle\int -\frac{dx}{\sqrt{1-x^2}} = \cos^{-1} x + C$	(Rearrangement of the above.)
$\dfrac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$	$\displaystyle\int \frac{dx}{1+x^2} = \tan^{-1} x + C$	$\int \frac{dx}{1+x^2} = \tan^{-1} x + C.$
$\dfrac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$	$\displaystyle\int -\frac{dx}{1+x^2} = \cot^{-1} x + C$	(Similar rearrangement.)
$\dfrac{d}{dx}(\sec^{-1} x) = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$\displaystyle\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + C$	(Sign depends on domain.)
$\dfrac{d}{dx}(\csc^{-1} x) = -\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$\displaystyle\int -\frac{dx}{x\sqrt{x^2-1}} = \csc^{-1} x + C$	(Similar domain detail.)
$\dfrac{d}{dx}(e^x) = e^x$	$\displaystyle\int e^x\,dx = e^x + C$	$\int e^x\,dx = e^x + C.$
$\dfrac{d}{dx}(\ln\lvert x\rvert) = \frac{1}{x}$	$\displaystyle\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \frac{dx}{x} = \ln\lvert x\rvert + C.$
$\dfrac{d}{dx}(a^x) = a^x \ln(a)$	$\displaystyle\int a^x\,dx = \frac{a^x}{\ln(a)} + C$	$\int 2^x\,dx = \frac{2^x}{\ln(2)} + C.$

2) Integration by Substitution (Selected Standard Results)

- $\displaystyle\int \tan x\,dx = \ln\lvert\sec x\rvert + C.$

  Example: $\int \tan x\,dx = \ln\lvert\sec x\rvert + C.$

- $\int \cot x\,dx = \ln|\sin x| + C.$
- $\int \sec x\,dx = \ln|\sec x + \tan x| + C.$
- $\int \csc x\,dx = \ln|\csc x - \cot x| + C.$

3) Integrals of Particular Forms

1. $\displaystyle\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C.$

   Example: $\int \frac{dx}{x^2-1} = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C.$

2. $\displaystyle\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + C.$
3. $\displaystyle\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C.$
4. $\displaystyle\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln\left|x + \sqrt{x^2 - a^2}\right| + C.$
5. $\displaystyle\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C.$
6. $\displaystyle\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left|x + \sqrt{x^2 + a^2}\right| + C.$

A More General Quadratic Denominator: $\displaystyle\int \frac{dx}{ax^2 + bx + c}.$

$$ax^2 + bx + c = a\left[x^2 + \frac{b}{a}x + \frac{c}{a}\right] = a\left[\left(x + \frac{b}{2a}\right)^2 + \left(\frac{c}{a} - \frac{b^2}{4a^2}\right)\right].$$

Then use the appropriate arctan or ln form depending on the sign of $\frac{c}{a} - \frac{b^2}{4a^2}$.

Simple Example:

$$\int \frac{dx}{x^2 + 1} = \tan^{-1}(x) + C,$$

which is the case $a = 1,\ b = 0,\ c = 1$.

Linear Numerator over Quadratic: $\displaystyle\int \frac{px + q}{ax^2 + bx + c}\,dx.$

Often one writes $\frac{px+q}{ax^2+bx+c}$ as

$$A \cdot \frac{d}{dx}\left(ax^2 + bx + c\right) + (\text{remainder}).$$

You then split it into a part that is the derivative of (ax^2+bx+c) plus a simpler leftover fraction.

Example: $\int \frac{2x}{x^2+1}\,dx = \int \frac{d(x^2+1)}{x^2+1} = \ln\left|x^2+1\right| + C.$

4) Rational-Function Integrals and Partial Fractions

When a rational function can be factored, we typically decompose it into simpler "partial fractions." Examples:

$$1.\ \frac{px+q}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b},$$
$$2.\ \frac{px+q}{(x-a)^2} = \frac{A}{x-a} + \frac{B}{(x-a)^2},$$
$$3.\ \frac{px+q+n}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c},$$
$$4.\ \frac{px+q+n}{(x-a)^2(x-b)} = \frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b},$$
$$5.\ \frac{px+q+n}{(x-a)\left(x^2+bx+c\right)} = \frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c}.$$

5) Integration by Parts

$$\int f(x)\,g(x)\,dx = f(x)\int g(x)\,dx - \int \left[f'(x)\int g(x)\,dx\right]dx.$$

Example: $\int x\,e^x\,dx$.

Let $f(x) = x \implies f'(x) = 1$, and $g(x) = e^x \implies \int g(x)\,dx = e^x$.

$$\int x\,e^x\,dx = xe^x - \int 1\cdot e^x\,dx = xe^x - e^x + C = e^x(x-1) + C.$$

6) Special Composite Forms

$$\int e^x\left[f(x) + f'(x)\right]dx = e^x\,f(x) + C,$$

because $\frac{d}{dx}\left[e^x f(x)\right] = e^x f'(x) + e^x f(x)$.

Example: $\int e^x(x^2+2x)\,dx$.

Notice $f(x) = x^2 \implies f'(x) = 2x$. Hence $x^2+2x = f(x) + f'(x)$. So

$$\int e^x(x^2+2x)\,dx = e^x\,x^2 + C.$$

7) Fundamental Theorems of Calculus

- **Area Function:** $A(x) = \int_a^x f(t)\,dt$.

If f is continuous on $[a, b]$, then $\frac{d}{dx}A(x) = f(x)$.

- **Second Part (Definite Integrals):**

  If $F'(x) = f(x)$, then
$$\int_a^b f(x)\,dx = F(b) - F(a).$$

8) Definite Integrals

- **Definition:** $\int_a^b f(x)\,dx = F(b) - F(a)$, where F is any antiderivative of f.
- **As a Limit of Sums:**

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\,\Delta x, \quad \Delta x = \frac{b-a}{n}.$$

9) Properties of Definite Integrals

Let f be integrable on $[a, b]$. Then:

P0. $\displaystyle\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx.$

P1. $\displaystyle\int_a^a f(x)\,dx = 0.$

P2. $\displaystyle\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx.$

Additional common properties:

- If f is even about $x = 0$, i.e. $f(-x) = f(x)$, then

$$\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx.$$

- If f is odd about $x = 0$, i.e. $f(-x) = -f(x)$, then

$$\int_{-a}^{a} f(x)\,dx = 0.$$

- If f has period a, then for any integer n:

$$\int_0^{na} f(x)\,dx = n\int_0^a f(x)\,dx.$$

(Optional) Some Other Standard Root Integrals

1. $$\int \sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\ln\left|x + \sqrt{x^2 - a^2}\right| + C.$$
2. $$\int \sqrt{x^2 + a^2}\,dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\ln\left|x + \sqrt{x^2 + a^2}\right| + C.$$
3. $$\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C.$$

Example: $\int \sqrt{x^2 + 4}\,dx = \frac{x}{2}\sqrt{x^2 + 4} + 2\ln\left|x + \sqrt{x^2 + 4}\right| + C.$

How to Use This Sheet:

1. Match the integrand to a known form (polynomials, trig, inverse trig, exponential, logarithmic, partial fractions, etc.).
2. Apply the relevant formula (or break down the problem).
3. Add the constant of integration ($+C$) for indefinite integrals.
4. For definite integrals, evaluate the antiderivative at the limits and subtract.

That covers the formulas and standard results with each formula translated into English and accompanied by a simple example.